Section P6– Factoring

1. **Definition**

To **factor** an algebraic expression we rewrite it as a product of factors using the distributive property

Factoring⟶
$$x^2 - 2x - 15 = (x+3)(x-5)$$
Sum of terms | product of factors
⟵Simplifying or expanding

2. **Factoring out Common Factors**

Each term of an expression is itself a product of factors. When the terms have a common factor, it can be factored out. $AB + AC = A(B+C)$

Examples: Factor out the common factor(s)

a) $-5x^3 + 10x$

b) $15x^3y - 5x^2y + 10x^2y^2$

c) $(3x+2)(x-3) + 5(x-3)^2$

3. **Factoring Binomials**

We can factor binomials as a product of two binomials in three cases:

Difference of two squares	$A^2 - B^2 =$
Difference of two cubes	$A^3 - B^3 =$
Sum of two cubes	$A^3 + B^3 =$
A and *B* being terms of the binomial	

Examples: Factor the binomial using a factoring formula

a) $4x^2 - 9$

b) $(x+2)^2 - (x-3)^2$

c) $8x^3 - 1$

d) $27x^6 + 1000$

4. <u>Factoring Expressions with Four Terms: Factoring by Grouping</u>

Examples: Factor the expression

a) $3x^3 - 6x^2 + 2x - 4$

b) $2x^5 - 10x^4 - x + 5$

5. <u>Factoring Trinomials</u>

5.1) <u>Trinomials of the form</u> $x^2 + bx + c$

If we can find two numbers r and s such that $rs = c$ and $r + s = b$ then we can rewrite directly $x^2 + bx + c$ as $(x + r)(x + s)$. If such numbers cannot be found then the trinomial does not factor.

Examples: Factor the trinomial

a) $3x - 10 + x^2$

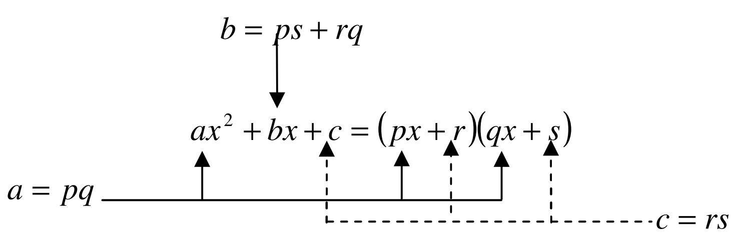
b) $x^6 - 2x^3 + 1$

c) $(x+1)^2 + 6(x+1) - 16$

d) $x^2 + 5yx + 6y^2$

5.2) <u>Trinomials of the form</u> $ax^2 + bx + c$, $a \neq 1$

We can use trial and error knowing that if $ax^2 + bx + c$ can be factored then we will have:



Or we can use the AC method (also called Key Number Method, or Master Product Method)

- Multiply together a and c.
- Find two numbers r and s such that $rs = ac$ and $r + s = b$. If such numbers cannot be found then the trinomial does not factor.
- Rewrite the middle term bx as $rx + sx$. We now have $ax^2 + rx + sx + c$
- Factor by grouping: $(ax^2 + rx) + (sx + c) = \ldots$

Examples: Factor the trinomial

a) $15 - 2x - x^2$

b) $2x^2 + 5x - 25$

c) $3x^2 - 5x - 2$

d) $6(x + y)^2 - 7(x + y) - 5$

5.3) <u>Recognizing a Perfect Square Trinomial</u>

A perfect square trinomial is the square of a binomial:

________________________________ $= (A + B)^2$
________________________________ $= (A - B)^2$

Examples: Identify the perfect square trinomial

a) $9x^2 + 30x + 25$

b) $36x^2 - 48x + 16$

6. Factoring an Expression Completely

When factoring an expression we need to check whether the factors in the factored form can be factored further.

WHEN FACTORING AN EXPRESSION ALWAYS CHECK FIRST WHETHER WE CAN FACTOR OUT A COMMON FACTOR

Examples: Factor the expression completely

a) $16x^5 - 2x^2$

b) $y^2(x^2 - 4) - 9(x^2 - 4)$

c) $x^3 + 5x^2 + 6x$

d) $(x-1)^3 - 2x(x-1)^2 + x^2(x-1)$

7. Factoring out a common variable factor in expressions with fractional exponents

The trick is to factor out the common factor that has the smallest exponent.

Examples: Factor the expression

a) $10x^{-1/2} + 7x^{1/2} + x^{3/2}$

b) $(x+2)^{8/3} - (x+2)^{2/3}$

c) $3x^{2/5}(x+1)^{3/5} + 2x^{7/5}(x+1)^{-2/5}$

Factoring

Greatest Common Factor: factor it out

- If there is a variable in each term with rational exponents, factor out the variable with ***lowest exponent***

Examples

$$2x^3 - 6x^2 + 4x = 2x\left(x^2 - 3x + 2\right)$$

$$x^{3/5} + 2x^{1/5} - 3x^{-2/5} = x^{-2/5}\left(x + 2x^{3/5} - 3\right)$$

Try factoring any expression that has:

- More than 1 term
- A power of the variable >1

Examples

2 terms

- (−)
  - $a^2 - b^2 = (a+b)(a-b)$
    - $x^2 - 4 = (x+2)(x-2)$
    - $9x^2 - 25 = (3x+5)(3x-5)$
    - $x^4 - 36 = \left(x^2+6\right)\left(x^2-6\right)$
  - $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$
    - $x^3 - 27 = (x-3)\left(x^2+3x+9\right)$
    - $64x^3 - 1 = (4x-1)\left(16x^2+4x+1\right)$
- (+)
  - $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$
    - $x^3 + 8 = (x+2)\left(x^2-2x+4\right)$
    - $27x^3 + 1000 = (3x+10)\left(9x^2-30x+100\right)$

3 terms → $ax^2 + bx + c$; Use AC method

- $2x^2 + 9x - 5 = 2x^2 + 10x - x - 5 = (2x-1)(x+5)$
- $x^4 + 3x^2 + 2 = \left(x^2+2\right)\left(x^2+1\right)$

4 terms → $rp + rq + sp + sq$; Factor by grouping

$$2x^3 - 4x^2 + 3x - 6 = 2x^2(x-2) + 3(x-2)$$
$$= \left(2x^2+3\right)(x-2)$$